Feb. 12, 2014

Slope Fields & Euler's Method

Slope Field / Euler's Method Worksheet

$-\frac{dy}{dx} = y - x - 1$ (Note that solns are $y = Ce^x + x + 2$
Don't need to know how to get this -
It's just an example
Start at $(-2, -1)$ - approx w/ Euler
Solution curve here is $y = -e^{x+2} + x + 2$

What is Euler's Method?

- Approximation method for estimating the solution to a differential equation.
- Use the idea of slope fields (go with the flow)
- The idea: start at some coordinate (a, b). Use diffy q to figure out what direction the curve will move. Move a little, and then look at diffy q again.

The steps:

Given: $\frac{dy}{dx} = F(x, y)$, $y(x_0) = y_0$ (this says that we are starting at (x_0, y_0))

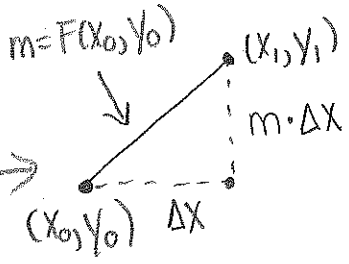
Want: an approximation for $y(x_n)$

x_n is some value we have picked.

What we do: First pick "step size"
How often are you going to refer back to $\frac{dy}{dx}$?
denote this h or Δx (change in x)

(1) Use $F(x,y)$ to take 1st step:
(x_1, y_1) will be next step

$$x_1 = x_0 + \Delta x$$
$$y_1 = y_0 + \Delta x \cdot F(x_0, y_0)$$



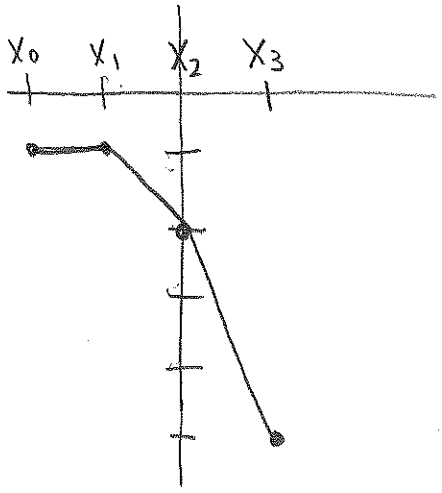
(2) Continue taking steps until you reach x_n.
In general:

$$x_{i+1} = x_i + \Delta x$$
$$y_{i+1} = y_i + \Delta x\, F(x_i, y_i)$$

Example: $\frac{dy}{dx} = y - x - 1 = F(x,y)$, start at $(-2, -1)$
Estimate $y(1)$

in this part let $\Delta x = 1$

i	x_i	y_i	$F(x_i, y_i)$	y_{i+1}
0	-2	-1	0	$-1 + 0 \cdot 1 = -1$
1	-1	-1	-1	$-1 + (-1)1 = -2$
2	0	-2	-3	$-2 + (-3) \cdot 1 = -5$
3	1	-5		



$y(1) \approx -5$
(actual value is: $-e^3 + 1 + 2 \approx -17.09$)

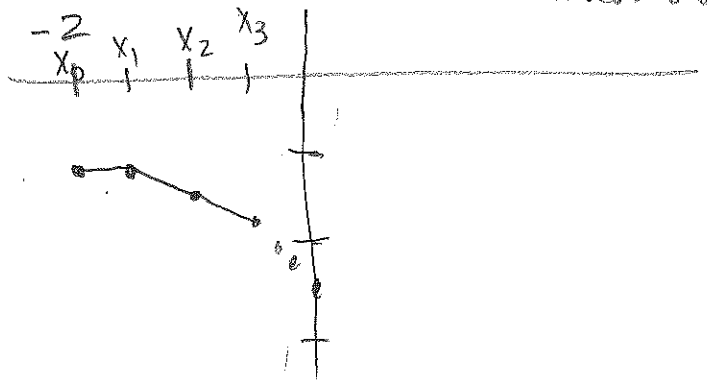
How do we get a better approximation?
Use a smaller value for Δx.

$\frac{dy}{dx} = y - x - 1$

Let $\Delta x = .5$

i	x_i	y_i	$F(x_i, y_i)$	y_{i+1}
0	-2	-1	0	-1
1	-1.5	-1	$-.5$	$-1+(-.5)(.5) = -1.25$
2	-1	-1.25	-1.25	$-1.25+(-1.25)(.5) = -1.875$
3	$-.5$	-1.875		
⋮				

It gets tedious pretty quickly. But the approx is closer to actual graph.



Slope Field and Euler Applet:

www.mathscoop.com/calculus/differential-equations/euler-method-calculator.php

Example: $\frac{dy}{dx} = \frac{y^2+3y}{x+4}$ and $y(0) = -1$.

Approximate $y(3)$, using $\Delta x = 1$

i	x_i	y_i	$F(x_i, y_i)$	y_{i+1}
0	0	-1	$\frac{1-3}{4} = -\frac{1}{2}$	$-1+(-\frac{1}{2})(1) = -\frac{3}{2}$
1	1	$-3/2$	$\frac{9/4 - 9/2}{5} = \frac{-9}{20}$	$-\frac{3}{2}+(\frac{-9}{20})(1) = \frac{-39}{20} \approx -1.95$
2	2	-1.95	$\frac{(-1.95)^2+3(-1.95)}{2+4} \approx -.34125$	$-1.95+(-.34125) = -2.29125$

$y(3) \approx -2.29125$